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DETERMINATION OF THERMAL WAVE DISTRIBUTIONS BY THE FINITE ELEMENT METHOD

M. D. MIKHAILOV,* G. COMINI,† S. DEL GIUDICE† and G. P. RUNCHI†

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Abstract—Quasi-stationary heat-conduction problems are formulated in such a way as to allow a direct solution by the finite element method without the use of recurrence relations. Accuracy and versatility of the technique proposed in the text are illustrated by several examples of application.

NOMENCLATURE

a,	thermal diffusivity [m^2/s];
Bi,	$= \alpha L/k$, Biot criterion;
c,	specific heat [$J/kg \cdot K$];
i,	$= \sqrt{(-1)}$, imaginary unit;
I,	imaginary part of complex amplitude θ [°C];
k,	thermal conductivity [$W/m \cdot K$];
l_x, l_y,	direction cosines of the outward normal to the boundary surface;
L,	slab thickness [m];
Pd,	$= \omega L^2/a$, Predvoditelev criterion;
r, z,	cylindrical coordinates [m];
R,	real part of complex amplitude θ [°C];
t,	time [s];
t_φ,	time lag of temperature oscillations [s];
T,	temperature [°C];
x, y,	Cartesian coordinates [m].

Greek symbols

α,	convective heat-transfer coefficient [$W/m^2 \cdot K$];
Γ,	boundary surface [m^2];
θ,	complex amplitude of temperature oscillations [°C];
ρ,	density [kg/m^3];
φ,	phase angle of temperature oscillations [rad];
ω,	circular frequency [rad/s];
Ω,	domain of definition [m^3].

Subscripts

a,	ambient;
x, y,	in the (x, y) direction;
w,	surface.

INTRODUCTION

SOLUTION of heat-conduction problems subjected to a boundary condition which is a periodic function of time is of interest in many fields of engineering. Applications of great practical importance are the calculations of heating and cooling loads in buildings exposed to periodic variations of the outside air temperature while the inside air temperature is maintained constant [1]. Examples of considerable significance occur also in the investigation of the oscillatory behaviour of internal combustion engine cylinder walls and in the study of the penetration of the daily and annual temperature cycles in the earth [2].

Usually the analytical solution of problems on thermal waves is given for the quasi-stationary state since, in most engineering applications, the process continues so long, or the transient decays so rapidly, that the initial temperature distribution has very little influence on the process behaviour [2–5]. In cases like these, to obtain only the sustained, periodic response of the system to a periodic disturbance allows a great saving in time and effort at the expense of no significant loss in accuracy.

The above arguments apply even more to problems where complex geometrical configurations and/or the presence of composite regions make recourse to numerical solutions mandatory. In fact, both finite difference and finite element solutions of initial value problems involve step-by-step procedures in which recurrence relations are used to move ahead in time. Therefore, if only the quasi-stationary state is of interest, calculations must be repeated for the very large number of successive time intervals required to reach the steady-periodic condition.

In this paper steady-state problems involving periodic boundary conditions are formulated in such a way as to allow a direct numerical solution without the use of recurrence relations. In the following sections finite element applications are stressed, but extensions to finite difference calculations are straightforward. Besides, minor modifications in the solution procedure would allow dealing with quasi-stationary fields produced by periodic internal heat generation.

Despite their practical importance, the problems considered here have received, until now, very little attention. A direct determination of periodic structural response has been proposed by Zienkiewicz in the context of finite elements ([6] p. 347). However, in [6] reference is made chiefly to linear vibration theory and the corresponding formulation for heat conduction is invalidated by an algebraic error.

MATHEMATICAL MODEL

The general class of problems dealt with in this paper can be described, in a two-dimensional region Ω, by

*Applied Mathematics Centre, VMEI, Sofia, Bulgaria.
†Istituto di Fisica Tecnica dell' Università e Laboratorio per la Tecnica del Freddo del C.N.R., via Marzolo 9, 35100 Padova, Italy.

the following equation:

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k_x \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial T}{\partial y}\right). \quad (1)$$

Boundary conditions:

$$T = T_W \quad (2)$$

on part of boundary Γ_1 and:

$$k_x \frac{\partial T}{\partial x} l_x + k_y \frac{\partial T}{\partial y} l_y + \alpha(T - T_a) = 0 \quad (3)$$

on part of boundary Γ_2 are assumed, where T_W and T_a are periodic functions of time of the form:

$$T_W = \theta_W \exp(i\omega t) \quad (4)$$

$$T_a = \theta_a \exp(i\omega t). \quad (5)$$

Then the quasi-stationary field can be described, throughout Ω, by the expression:

$$T(x, y, t) = \theta(x, y)\exp(i\omega t). \quad (6)$$

When no restriction is placed on the shape of the periodic variation, the present analysis holds still good for every harmonic and the general solution can be obtained by superposition of the different modes.

In equations (4)–(6) $\theta(x, y)$ is a complex function:

$$\theta = R + iI \quad (7)$$

whose modulus $|\theta| = (R^2 + I^2)^{1/2}$ and argument $\varphi = \arctan(I/R)$ give, respectively, the amplitude and the phase angle of temperature oscillations.

Substitution of (7) into (1), (2), (3) leads to:

$$\frac{\partial}{\partial x}\left(k_x \frac{\partial \theta}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial \theta}{\partial y}\right) - i\omega\rho c\theta = 0 \quad (8)$$

and:

$$\theta = \theta_W \quad \text{on} \quad \Gamma_1, \quad (9)$$

$$k_x \frac{\partial \theta}{\partial x} l_x + k_y \frac{\partial \theta}{\partial y} l_y + \alpha(\theta - \theta_a) = 0 \quad \text{on} \quad \Gamma_2. \quad (10)$$

If reference is made separately to the real and imaginary parts of θ, we obtain the following system of equations:

$$\left.\begin{aligned} \frac{\partial}{\partial x}\left(k_x \frac{\partial R}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial R}{\partial y}\right) + \omega\rho c I = 0 \\ \frac{\partial}{\partial x}\left(k_x \frac{\partial I}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial I}{\partial y}\right) - \omega\rho c R = 0 \end{aligned}\right\} \quad (11)$$

subjected to boundary conditions:

$$R = R_W, \qquad I = I_W \quad (12)$$

on Γ_1 and:

$$\left.\begin{aligned} k_x \frac{\partial R}{\partial x} l_x + k_y \frac{\partial R}{\partial y} l_y + \alpha(R - R_a) = 0 \\ k_x \frac{\partial I}{\partial x} l_x + k_y \frac{\partial I}{\partial y} l_y + \alpha(I - I_a) = 0 \end{aligned}\right\} \quad (13)$$

on Γ_2.

A few analytical solutions of problems (11)–(13) are available for one-dimensional geometries [2–5]. In the general case, where composite regions and/or complex geometrical configurations have to be dealt with, recourse must be made to numerical solutions.

FINITE ELEMENT FORMULATION

The unknown functions R and I can be approximated, throughout the solution domain Ω, by the relationships:

$$R \cong \sum_{j=1}^{n} N_j(x, y)R_j = [N]\{R\} \quad (14)$$

and

$$I \cong \sum_{j=1}^{n} N_j(x, y)I_j = [N]\{I\} \quad (15)$$

where N_j, or $[N]$, are the usual shape functions defined piecewise element by element, R_j and I_j, or $\{R\}$ and $\{I\}$, being the nodal parameters [6–9].

The $2n$ simultaneous equations allowing the solution for n values of R_j and n values of I_j are obtained using Galerkin's method as shown in [7–9]. Typically, for point $j (j = 1, n)$, the integrals over the domain Ω of the products of the weighting function N_j by the residual resulting from substitution of equations (14), (15) into (11) are equated to zero:

$$\left.\begin{aligned} \int_\Omega N_j\left\{\left[\frac{\partial}{\partial x}\left(k_x \frac{\partial}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial}{\partial y}\right)\right]\sum_{k=1}^{n} N_k R_k \right. \\ \left. + \omega\rho c \sum_{k=1}^{n} N_k I_k\right\} d\Omega = 0 \\ \int_\Omega N_j\left\{\left[\frac{\partial}{\partial x}\left(k_x \frac{\partial}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial}{\partial y}\right)\right]\sum_{k=1}^{n} N_k I_k \right. \\ \left. - \omega\rho c \sum_{k=1}^{n} N_k R_k\right\} d\Omega = 0 \end{aligned}\right\}. \quad (16)$$

After using Green's theorem, in order to avoid second derivatives in the integrals imposing unnecessary continuity conditions between elements, equations (16) are transformed into

$$\left.\begin{aligned} \int_\Omega \left(\frac{\partial N_j}{\partial x} \sum_{k=1}^{n} k_x \frac{\partial N_k}{\partial x} + \frac{\partial N_j}{\partial y} \sum_{k=1}^{n} k_y \frac{\partial N_k}{\partial y}\right) R_k \, d\Omega \\ + \int_\Gamma N_j \alpha\left(\sum_{k=1}^{n} N_k R_k - R_a\right) d\Gamma \\ - \omega \int_\Omega N_j \rho c \sum_{k=1}^{n} N_k I_k \, d\Omega = 0 \\ - \int_\Omega \left(\frac{\partial N_j}{\partial x} \sum_{k=1}^{n} k_x \frac{\partial N_k}{\partial x} + \frac{\partial N_j}{\partial y} \sum_{k=1}^{n} k_y \frac{\partial N_k}{\partial y}\right) I_k \, d\Omega \\ - \int_\Gamma N_j \alpha\left(\sum_{k=1}^{n} N_k I_k - I_a\right) d\Gamma \\ - \omega \int_\Omega N_j \rho c \sum_{k=1}^{n} N_k R_k \, d\Omega = 0 \end{aligned}\right\}. \quad (17)$$

where $(j = 1, n)$.

Equations (17) can be written in matrix form as:

$$\begin{vmatrix} [K] & -\omega[C] \\ -\omega[C] & -[K] \end{vmatrix} \begin{Bmatrix} \{R\} \\ \{I\} \end{Bmatrix} = \begin{Bmatrix} \{FR\} \\ -\{FI\} \end{Bmatrix}. \quad (18)$$

Typical matrix elements are:

$$K_{jk} = \sum \int_{\Omega^e} \left(\frac{\partial N_j}{\partial x} k_x \frac{\partial N_k}{\partial x} + \frac{\partial N_j}{\partial y} k_y \frac{\partial N_k}{\partial y} \right) \mathrm{d}\Omega + \sum \int_{\Gamma^e} N_j \alpha N_k \,\mathrm{d}\Omega; \qquad (19)$$

$$C_{jk} = \sum \int_{\Omega^e} N_j \rho c N_k \,\mathrm{d}\Omega; \qquad (20)$$

$$FR_j = \sum \int_{\Gamma^e} N_j \alpha R_a \,\mathrm{d}\Gamma; \qquad (21)$$

$$FI_j = \sum \int_{\Gamma^e} N_j \alpha I_a \,\mathrm{d}\Gamma \qquad (22)$$

where $(j, k = 1, n)$.

In the above the summations are taken over the contributions of each element, Ω^e is the element region and Γ^e refers only to elements with external boundaries on which conditions (13) are specified.

The coupled system represented by equation (18) is a symmetric, "two degrees of freedom" system.

The FORTRAN program for implementing and solving the system of equations (18) is the same, but for minor modifications, which is referred to in [8, 9] where different coupled problems are investigated.

Isoparametric elements, numerical integration of equations (19)–(22) and a Gaussian elimination technique in the inversion of the "stiffness" matrix at the left hand side of equation (18) are all typical features of the code utilized [8, 9].

The program can also deal with axialsymmetric problems by assuming: $x \equiv r$, $y \equiv z$ and utilizing "equivalent" values of thermal properties and transfer coefficients: $k_{eq} = rk$, $\rho c_{eq} = r\rho c$, $\alpha_{eq} = r\alpha$ (see for instance [6] p. 302; [10] p. 252).

Output of data has been modified to include, for each node, values of R, I, modulus: $(R^2 + I^2)^{1/2}$, phase: $\arctan(I/R)$ and time lag: t_φ of temperature oscillations.

At the nodes where boundary conditions (2) or (3) are specified, amplitude and time lag of heat flux oscillations are also computed.

SOME ILLUSTRATIVE EXAMPLES

The first two examples are of a comparative nature and deal with simple configurations for which analytical solutions exist. The last three examples are more general and illustrate practical applications.

Thermal waves in infinite plates

Analytical solutions for quasi-stationary one-dimensional heat-conduction processes, subjected to boundary conditions which are periodic functions of time, have been obtained by several authors (see, for example [4, 5]).

In the present context these problems have been solved with reference to a rectangular configuration whose boundaries are non-conductive except at the face $x = 0$, where boundary conditions (2) or (3) apply.

Comparisons with analytical solutions are presented in Table 1 for the case:

$$a = k/\rho c = 1\,\mathrm{m^2/s}, \quad \omega = 1\,\mathrm{rad/s}, \quad L = 10\,\mathrm{m},$$
$$\text{i.e. } Pd = \omega L^2/a = 100.$$

Reference is made to boundary conditions of the first kind: $|\theta_W| = 1°\mathrm{C}$, $\varphi_W = 0$ and to boundary conditions of the third kind: $|\theta_a| = 1°\mathrm{C}$, $\varphi_a = 0$, $\alpha = 1\,\mathrm{W/m^2 \cdot K}$, i.e. $Bi = \alpha L/k = 10$.

Despite the rather coarse mesh utilized (5 parabolic elements, 28 nodal points) agreement between computed and analytical solutions is quite good.

Thermal waves in a composite junction

Equipment selection for air conditioning systems is often based on the assumption of maximum cooling loads occurring at the same time in all conditioned spaces. This approach was developed when all design calculations were performed manually. Now with computers used for routine design calculations, it is practical to make a more extensive design analysis. For example, heat gains by conduction through one-dimensional composite walls can be computed, for variable operating conditions, utilizing the transfer function method as suggested in [1].

However, one-dimensional models cannot always represent adequately civil engineering structures. Prefabrication techniques and the consequent large scale utilization of the same components in different buildings often make a finite element analysis of periodic thermal fields the most economic choice.

To illustrate the possibilities available in this type of analysis, amplitude and time lag distributions of temperature oscillations in a composite junction are first investigated.

The mesh utilized is represented in Fig. 1: 16 parabolic elements and 67 nodal points are used. Far away boundaries are substituted with non conductive boundaries placed at a reasonable distance from the junction. Because of the existing symmetry reference can be made only to a half of the entire domain.

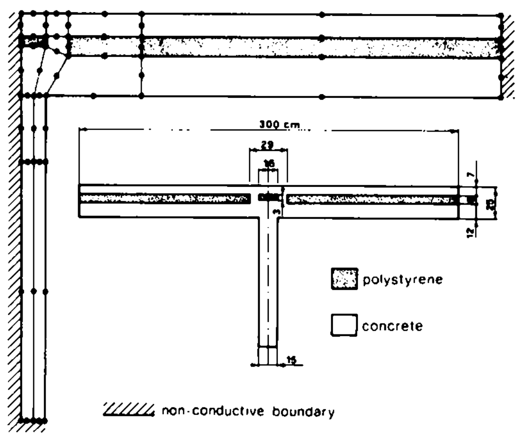

FIG. 1. Thermal waves in a composite junction. Far away boundaries are substituted with non-conductive boundaries placed at a relatively large distance from the junction. Because of the existing symmetry only half of the domain is investigated using 16 parabolic elements and 67 nodal points.

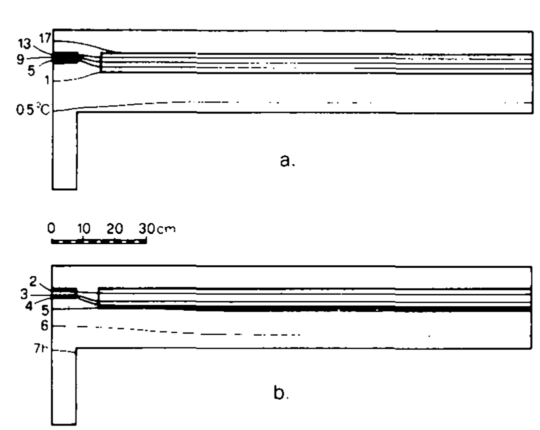

Fig. 2. Thermal waves in a composite junction. Convective heat transfer is assumed both at the internal and at the external surface: internal surface: $\alpha = 15\,W/m^2 \cdot K$, $|\theta_a| = 0°C$; external surface: $\alpha = 25\,W/m^2 \cdot K$, $|\theta_a| = 20°C$, $\varphi_a = 0$, $\omega = 7.27 \times 10^{-5}$ rad/s.
The following estimates of thermal properties are made: concrete: $k_x = k_y = 1.75\,W/m \cdot K$, $\rho c = 1.85 \times 10^6\,J/m^3 \cdot K$; polystyrene: $k_x = k_y = 0.039\,W/m \cdot K$, $\rho c = 1.6 \times 10^4\,J/m^3 \cdot K$. (a) Amplitude of temperature oscillations. (b) Time lag of temperature oscillations.

Convective heat transfer is assumed both at the external and at the internal surface and the following estimates of physical parameters are made:

- external surface: $\alpha = 25\,W/m^2 \cdot K$, $|\theta_a| = 20°C$, $\varphi_a = 0$, $\omega = 7.27 \times 10^{-5}$ rad/s (i.e. $2\pi/\omega \cong 24\,h$);
- internal surface: $\alpha = 15\,W/m^2 \cdot K$, $|\theta_a| = 0°C$,
- concrete: $k_x = k_y = 1.75\,W/m \cdot K$, $\rho_c = 1.85 \times 10^6\,J/m^3 \cdot K$;
- polystyrene: $k_x = k_y = 0.039\,W/m \cdot K$, $\rho_c = 1.6 \times 10^4\,J/m^3 \cdot K$.

The amplitude of temperature oscillations at any point of the domain is shown in Fig. 2(a) while time lags are represented in Fig. 2(b).

Amplitude and time lag of heat flux oscillations were also calculated at nodal points on the surfaces with convective boundary conditions specified.

In regions where one-dimensional fields develop, there exists good agreement between the values of temperature and heat flux thus calculated and the analytical solutions available for one-dimensional composite layers [1, 5].

Thermal waves in corner junctions

Finite element analyses of thermal wave distributions in building components are used, in the following example, to illustrate the effects of thermal bridges on heat gains by conduction through multi-layered exterior walls.

Here we investigate the thermal behaviour of two different corner junctions. In the first construction only concrete is utilized in the corner region. In the second construction instead, polystyrene insulation is not interrupted in the corner region.

In both cases the same mesh, with 15 parabolic elements and 62 nodal points, is used (see Fig. 3). The same values of thermal properties and transfer parameters referred to in the previous example are chosen. In particular we have again:

external surface: $\alpha = 25\,W/m^2 \cdot K$, $|\theta_a| = 20°C$, $\varphi_a = 0$, $\omega = 7.27 \times 10^{-5}$ rad/s;

internal surface: $\alpha = 15\,W/m^2 \cdot K$, $|\theta_a| = 0°C$.

The results obtained are reported in Fig. 4. As it can be seen, the amplitude of temperature oscillations in the corner region is dramatically reduced by the polystyrene insulation. As a consequence, the amplitude of heat flux oscillations at the internal edge is reduced by a factor of five with respect to the completely insulated construction.

Amplitude and time lag of temperature oscillations can be determined also using finite elements as suggested in [7] for transient field problems.

Obviously, step-by-step procedures are not a good choice in the solution of quasi-stationary problems. However, in the present context they allow a comparison of well established numerical techniques with the direct method proposed in this paper.

Reference was made to the not completely insulated construction and calculations were carried on for three one day temperature cycles, starting from constant temperature values and using 300 s time-steps. After 48 h, initial conditions do not have any influence on

Table 1. Analytical (AN) and finite element (FE) solutions compared for boundary conditions of the first kind: $|\theta_w| = 1°C$, $\varphi_w = 0$ and $Pd = \omega L^2/a = 100$; and third kind: $|\theta_a| = 1°C$, $\varphi_a = 0$, $Bi = \alpha L/k = 10$ and $Pd = 100$

	1st kind b.c.				3rd kind b.c.			
x/L	$\lvert\theta\rvert$ (°C)		φ (rad)		$\lvert\theta\rvert$ (°C)		φ (rad)	
	AN	FE	AN	FE	AN	FE	AN	FE
0.0	1.00000	1.00000	0.00000	0.00000	0.54120	0.54424	0.39270	0.39264
0.1	0.49307	0.49453	0.70711	0.70653	0.26685	0.26914	1.0998	1.0992
0.2	0.27312	0.24016	1.7172	1.4333	0.13157	0.13070	1.8069	1.8259
0.3	0.11987	0.11876	2.1195	2.1398	0.06487	0.06463	2.5140	2.5324
0.4	0.05910	0.05767	2.8293	2.8667	0.03198	0.03138	3.2213	3.2593
0.5	0.02916	0.02854	3.5361	3.5737	0.01578	0.01553	3.9288	3.9663
0.6	0.01441	0.01389	4.2406	4.2981	0.00780	0.00756	4.6333	4.6907
0.7	0.00704	0.00681	4.9368	4.9936	0.00381	0.00371	5.3296	5.3862
0.8	0.00330	0.00314	5.6761	5.7497	0.00178	0.00171	6.0688	6.1423
0.9	0.00184	0.00174	6.5913	6.6660	0.00099	0.00095	6.9840	7.0586
1.0	0.00170	0.00123	7.0711	7.1664	0.00092	0.00087	7.4638	7.5590

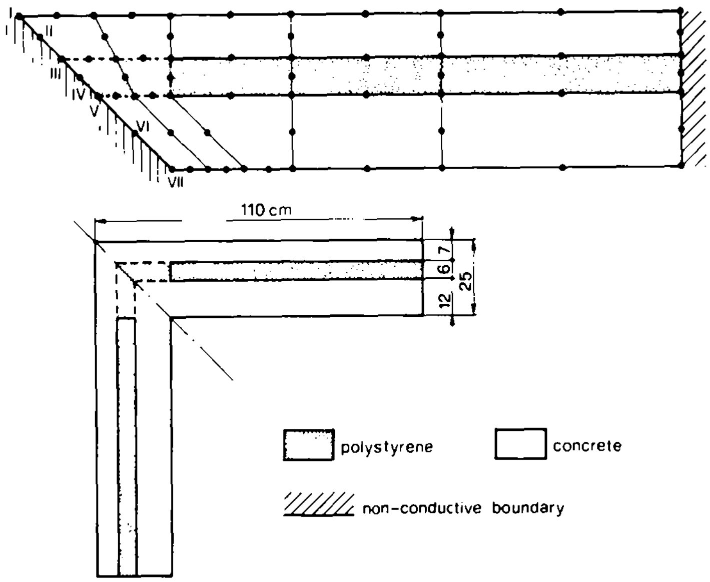

FIG. 3. Thermal waves in a corner junction. The same finite element mesh (15 parabolic elements and 62 nodal points) is used to study both the insulated and the not completely insulated construction.

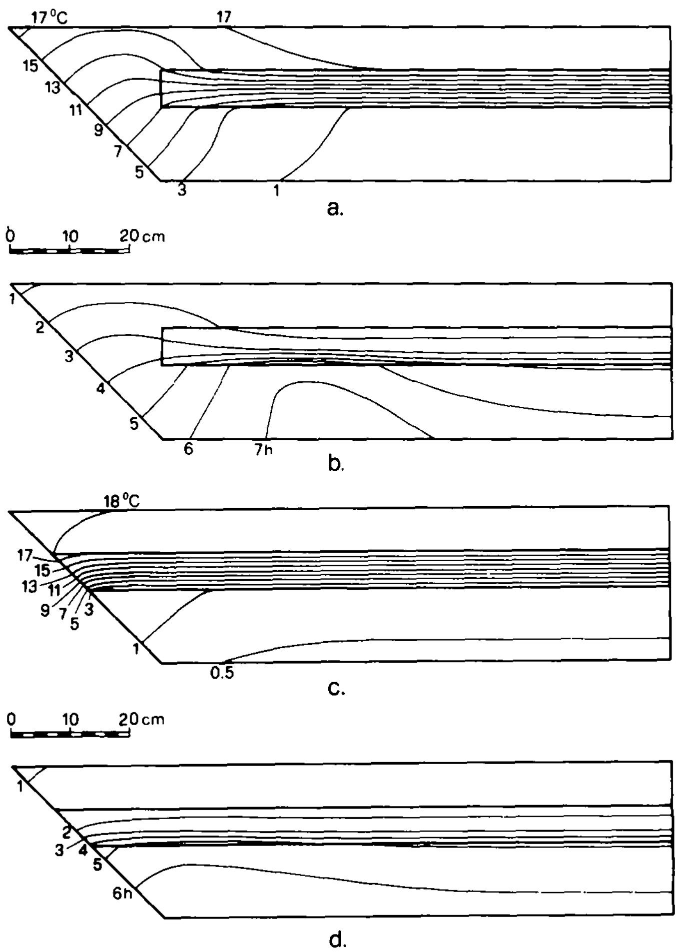

FIG. 4. Thermal waves in a corner junction. Boundary conditions and thermal property values are the same referred to in Fig. 2 (a) and (b) Amplitude and time lag of temperature oscillations in the not completely insulated construction. (c) and (d) amplitude and time lag of temperature oscillations in the insulated construction.

the first three significant figures of computed temperature values. Thus from that time, amplitude and time lag of temperature oscillations can be confidently compared with those obtained from the direct solution. The results are presented in Table 2, where the representative points I–VII shown in Fig. 3 are considered. As it can be seen, agreement is exceedingly good.

Table 2. Standard step-by-step (*S*) and direct (*D*) finite element solutions are compared for the representative points I–VII shown in Fig. 3. The not completely insulated corner junction is considered

	$\|\theta\|$ (°C)		t_φ(s)	
Point	*S*	*D*	*S*	*D*
I	17.97	17.96	2328	2327
II	15.97	15.97	5014	5013
III	14.11	14.11	7808	7807
IV	12.50	12.50	10 168	10 167
V	10.91	10.91	12 331	12 329
VI	7.103	7.102	16 483	16 480
VII	3.710	3.711	20 212	20 209

Comparison of computing times is very much in favour of the direct solution technique that uses about one tenth of the CPU time required by the step-by-step procedure.

CONCLUSIONS

Quasi-stationary problems involving periodic boundary conditions can be formulated in such a way as to allow a direct solution without the use of recurrence relations.

Accuracy and versatility of this technique has been demonstrated in the context of finite element analyses with reference to several applications of great scientific and practical interest.

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DETERMINATION DES DISTRIBUTIONS D'ONDE THERMIQUE PAR LA METHODE DES ELEMENTS FINIS

Résumé—Des problèmes de conduction thermique quasi stationnaire sont formulés de telle sorte qu'une solution est obtenue directement par la méthode des éléments finis sans le recours à des relations de récurrence. La précision et la flexibilité de la technique proposée sont illustrées par différents exemples d'application.

BESTIMMUNG WELLENFÖRMIGER TEMPERATURVERTEILUNGEN MIT HILFE DER METHODE FINITER ELEMENTE

Zusammenfassung—Es werden quasi-stationäre Wärmeleitprobleme so formuliert, daß eine direkte Lösung mit Hilfe der Methode finiter Elemente möglich ist, ohne daß Rekursionsformeln verwendet werden müssen. Die Genauigkeit und Vielseitigkeit der vorgeschlagenen Methode wird anhand mehrerer Anwendungsbeispiele aufgezeigt.

РАСЧЕТ РАСПРЕДЕЛЕНИЯ ТЕПЛОВОЙ ВОЛНЫ МЕТОДОМ КОНЕЧНЫХ ЭЛЕМЕНТОВ

Аннотация — Задачи квазистационарной теплопроводности сформулированы таким образом, что они допускают непосредственное решение методом конечного элемента без использования рекуррентных отношений. Точность и универсальность предложенного метода иллюстрируются на практических примерах.